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# On the role of localised post-buckling equilibria in axially compressed cylinders

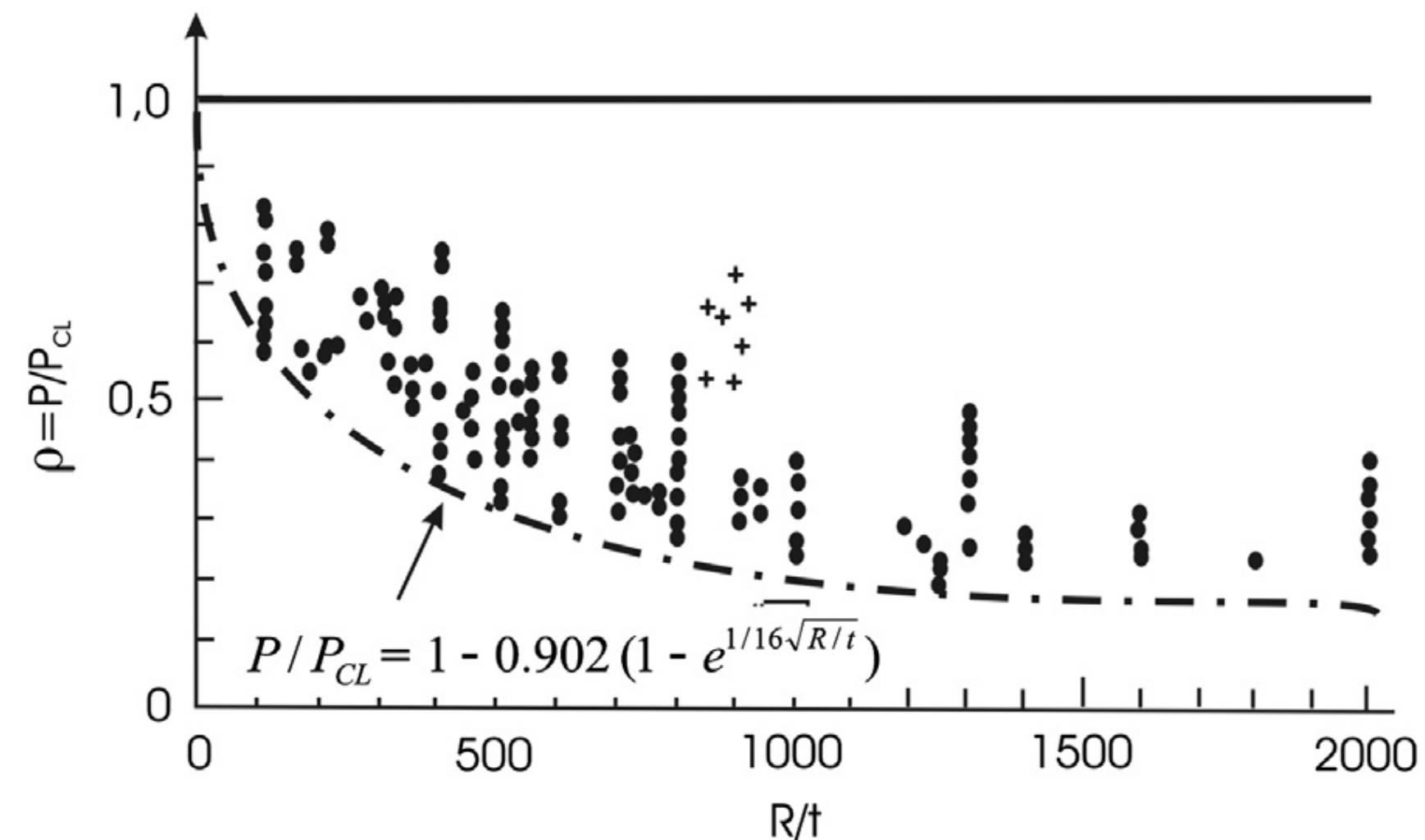
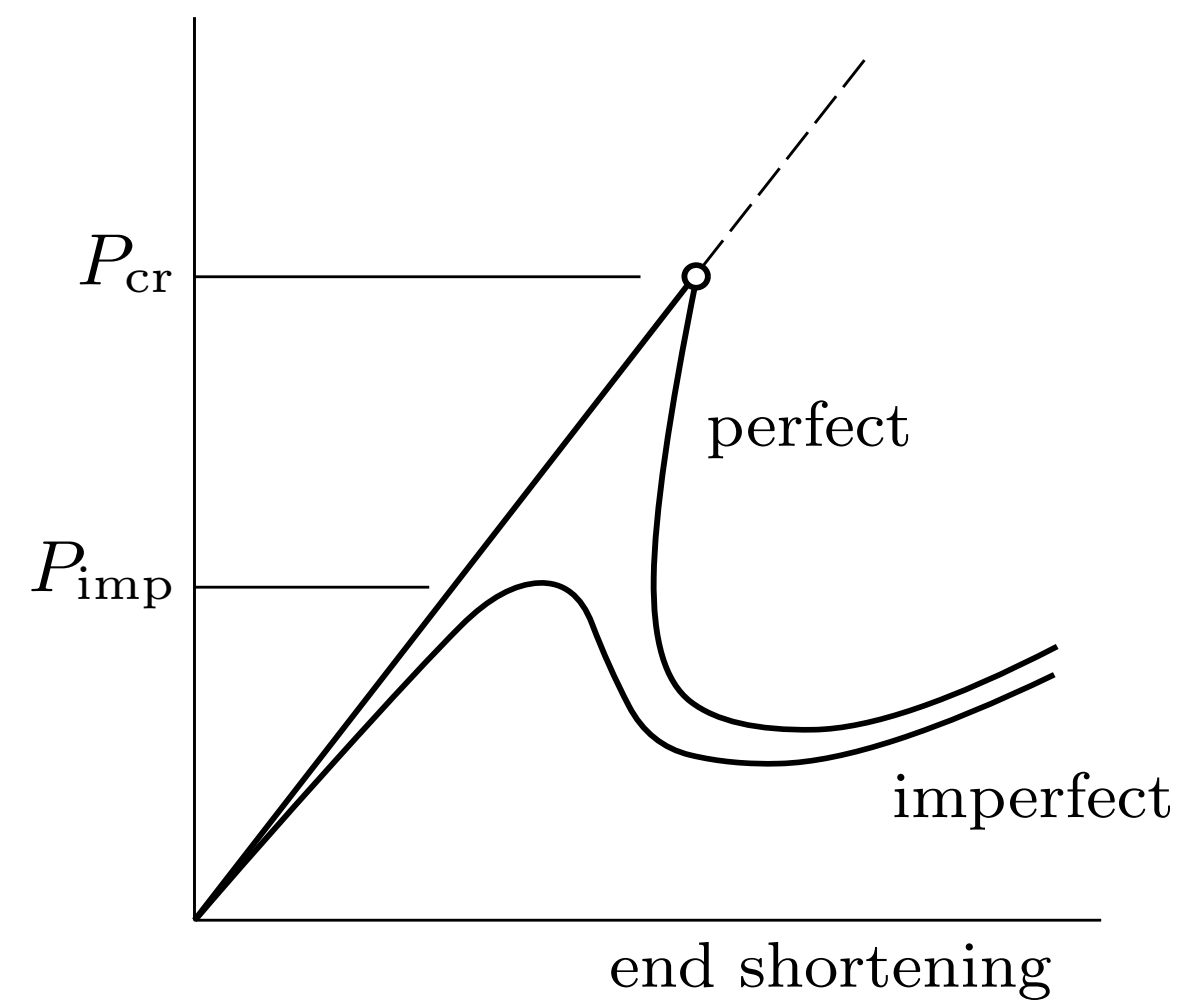
Rainer Groh & Alberto Pirrera

Boston, 8 March 2019



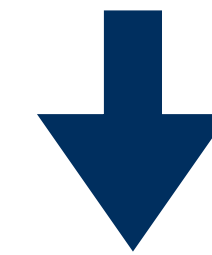
# Motivation

- Cylinder buckling governed by a subcritical bifurcation
  - leads to imperfection sensitivity
- Empirical (safe) design threshold, *e.g.* NASA SP-8007
  - too conservative for modern manufacturing methods
  - difficult for small satellite launchers
- Need less conservative knockdown factors



**Small satellite launchers  
*e.g.* Skyrora, Scotland**

**Smaller launchers are less  
efficient**



**Need less conservative  
knockdown factors**

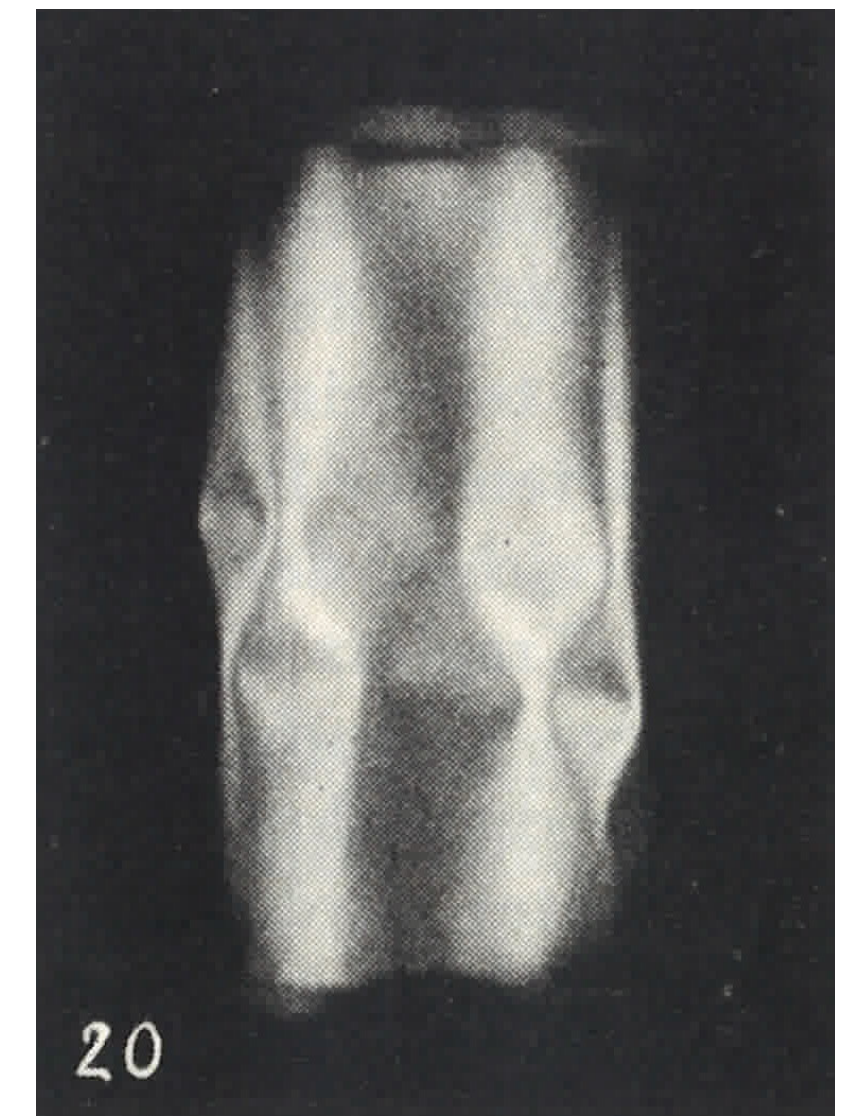
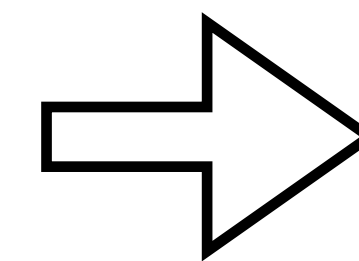
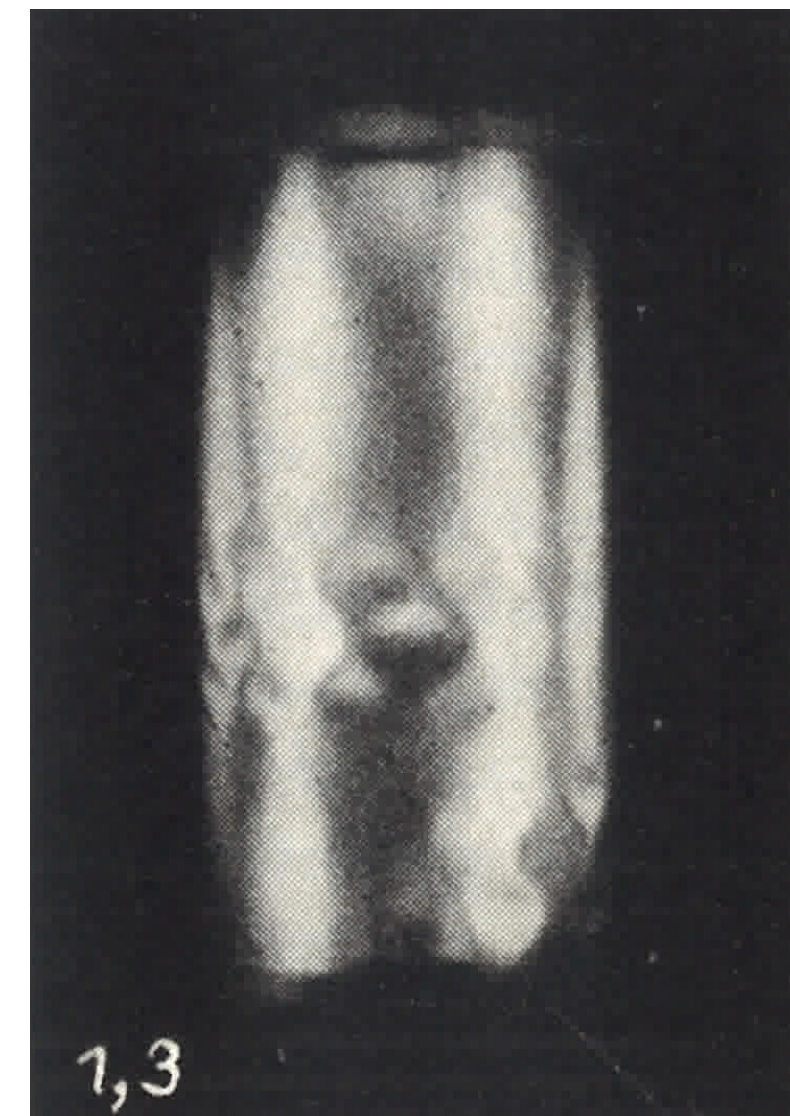
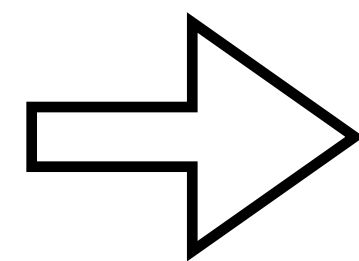
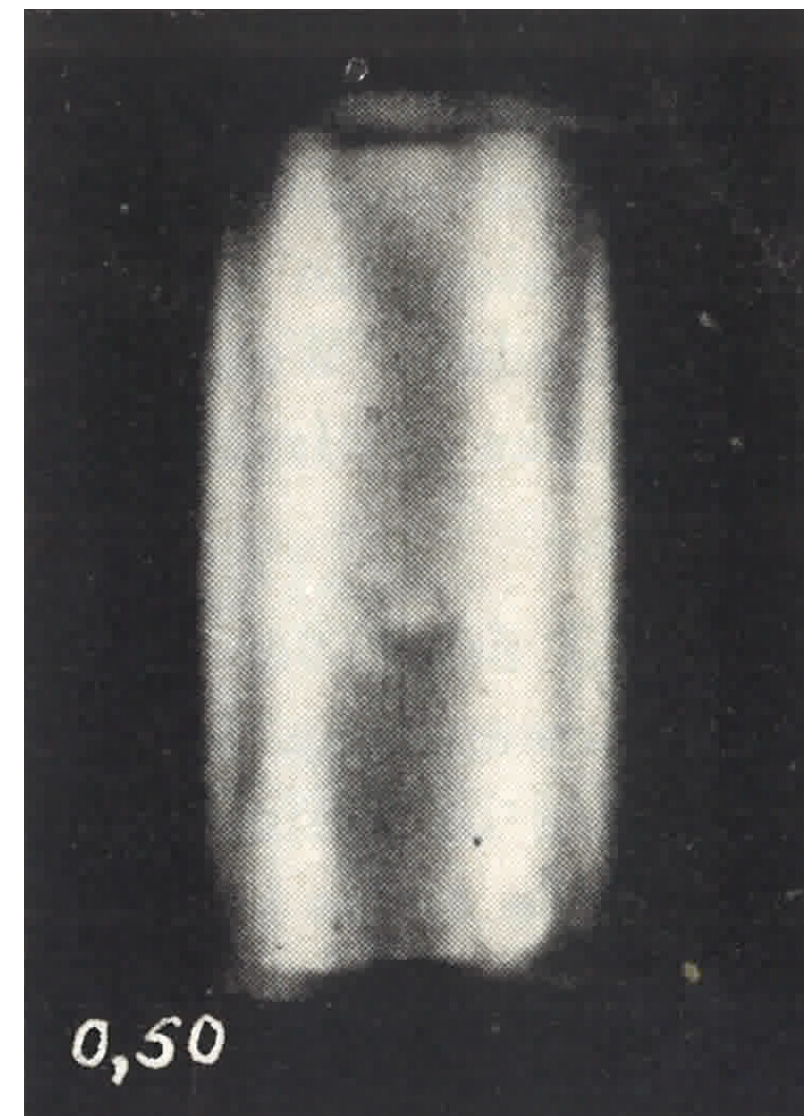




# Approach

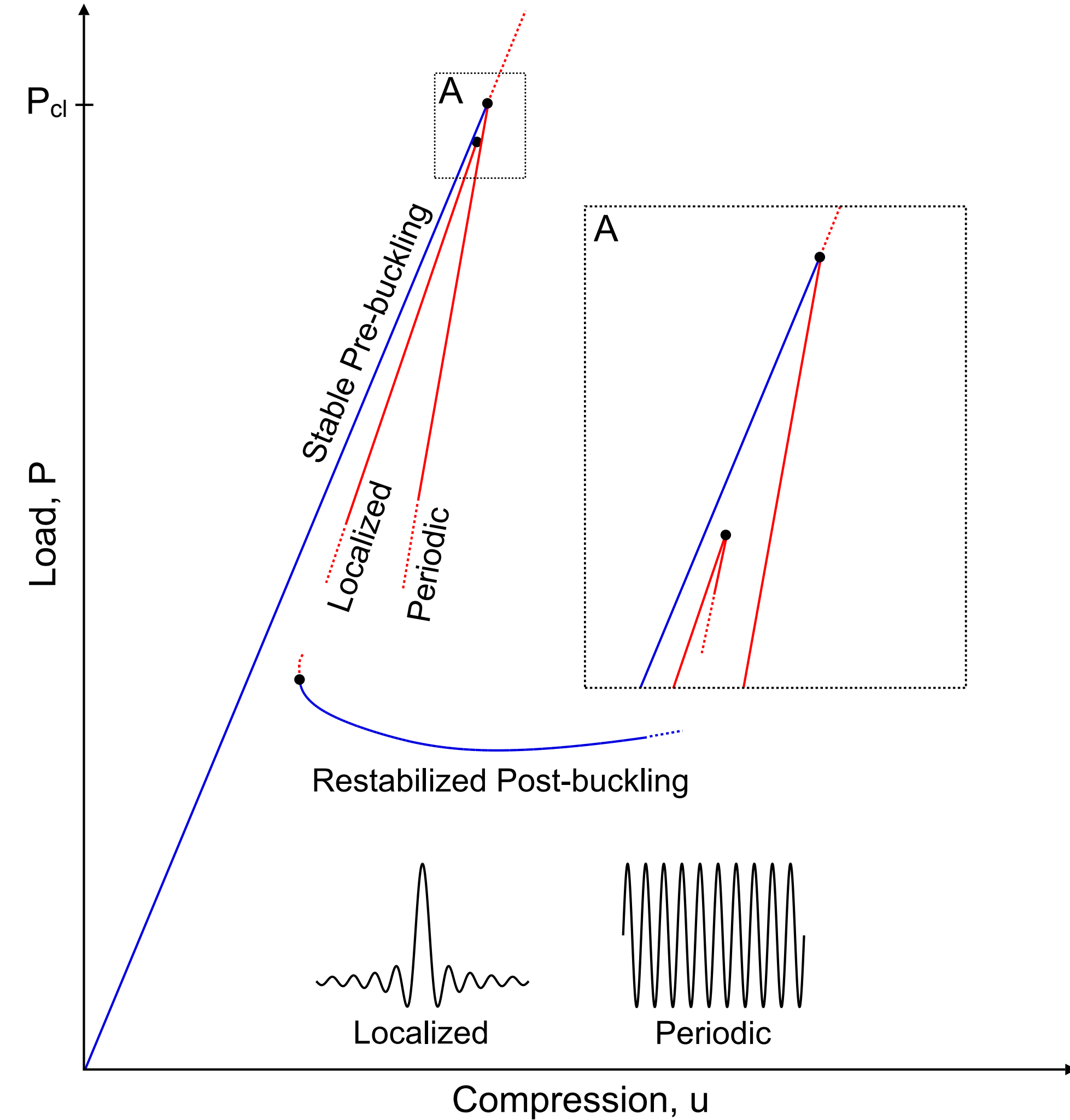
1. Study the equilibrium path of the buckling mode observed (most commonly) in experiments—the single dimple
2. Describe the deformation patterns that emerge as compression is varied
3. Derive a buckling threshold based on the level of compression for which the single dimple exists as an edge state

**Eßlinger & Geier  
(1972)**



# Generalised Path-Following

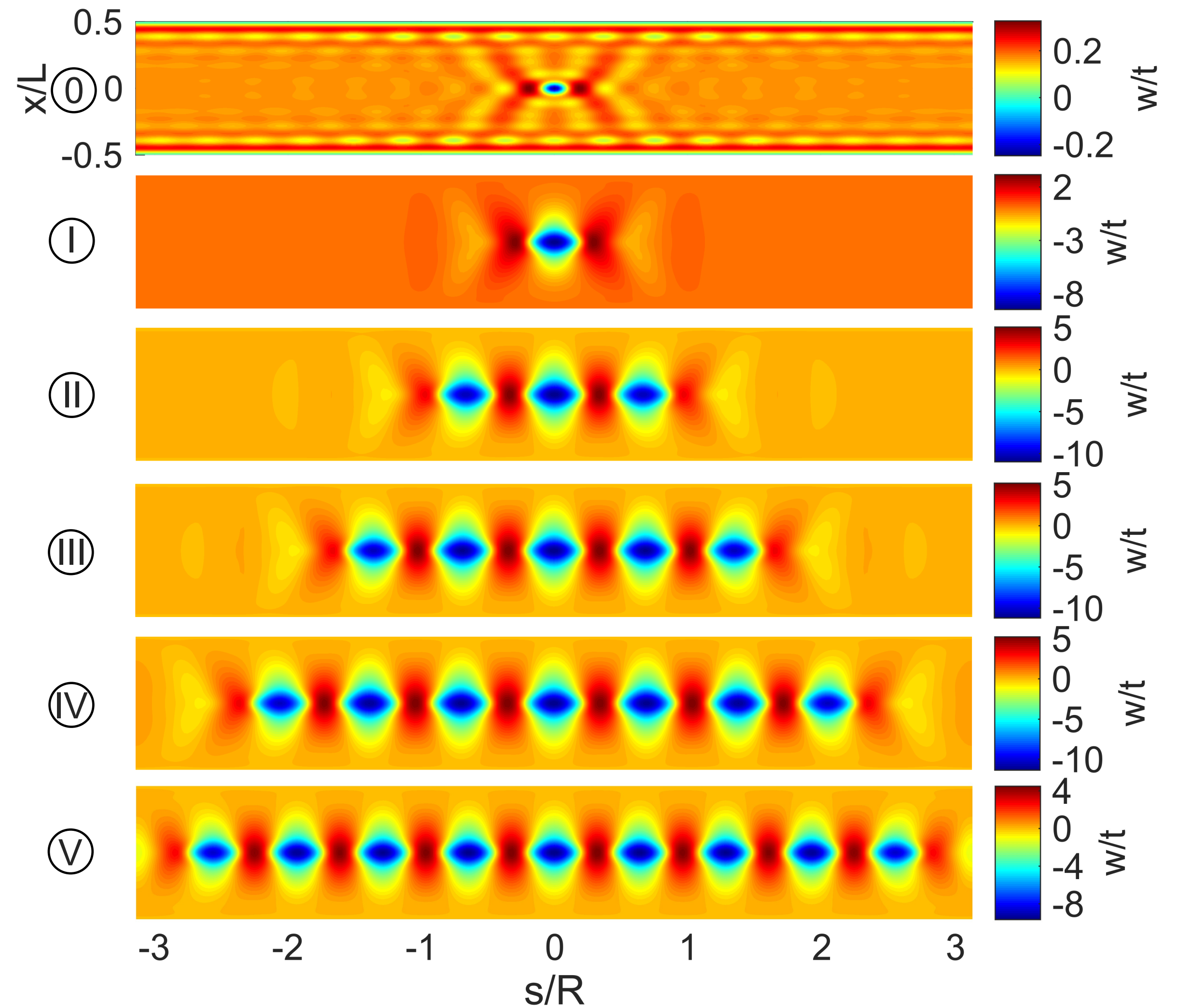
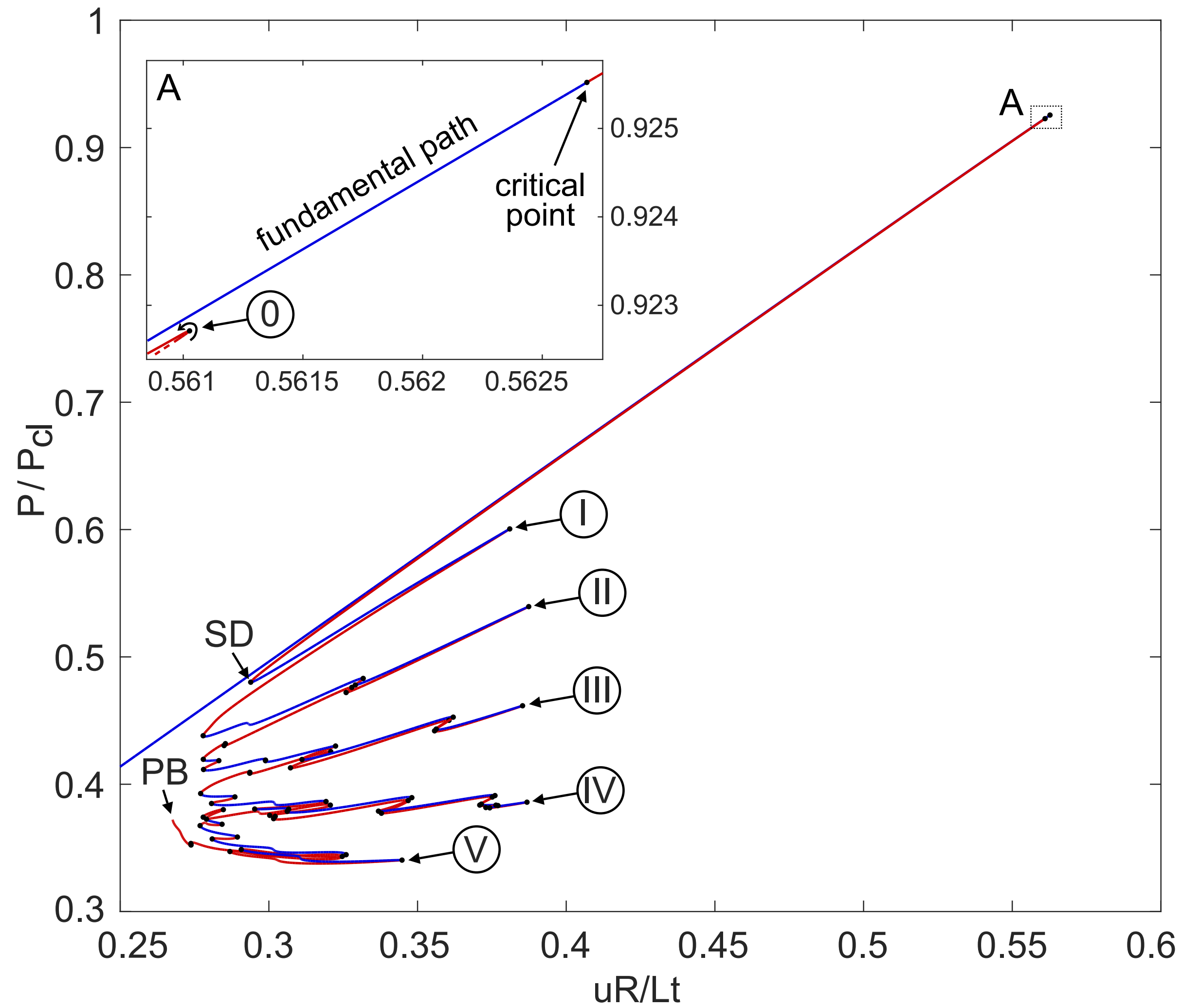
- “Riks” path-following with additional capabilities:
  - equilibrium is **stable** or **unstable**
  - pinpoint **critical points**
  - branch switch at bifurcation points
  - track critical points with other parameters
- Run Yamaki’s (1984) longest cylinder:
  - $R = 100$  mm,  $L = 160.9$  mm,  $t = 0.247$  mm
  - $E = 5.56$  GPa (Mylar),  $\nu = 0.3$
  - Both ends clamped + axial compression
  - Quarter model using nonlinear finite elements with large rotations



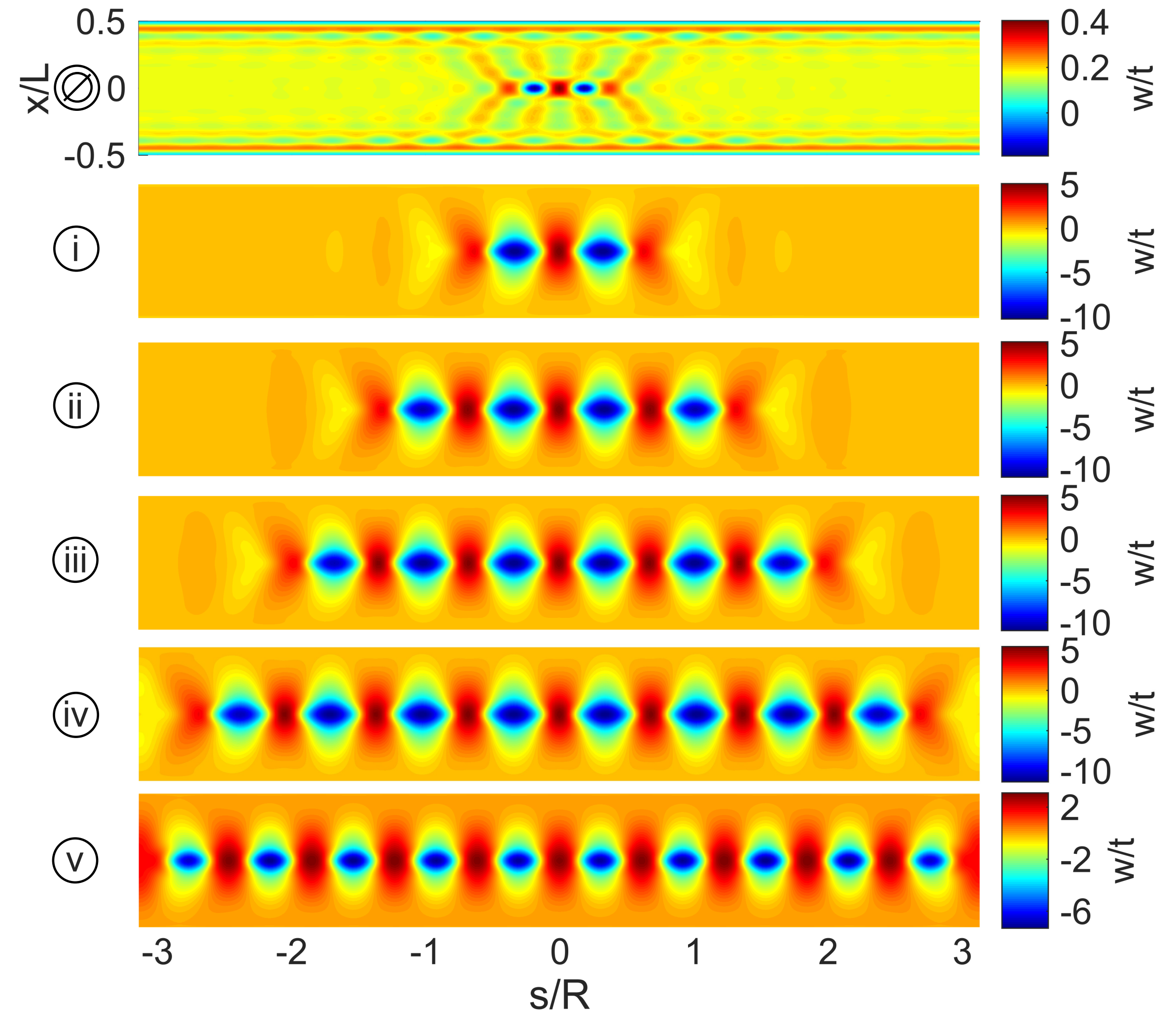
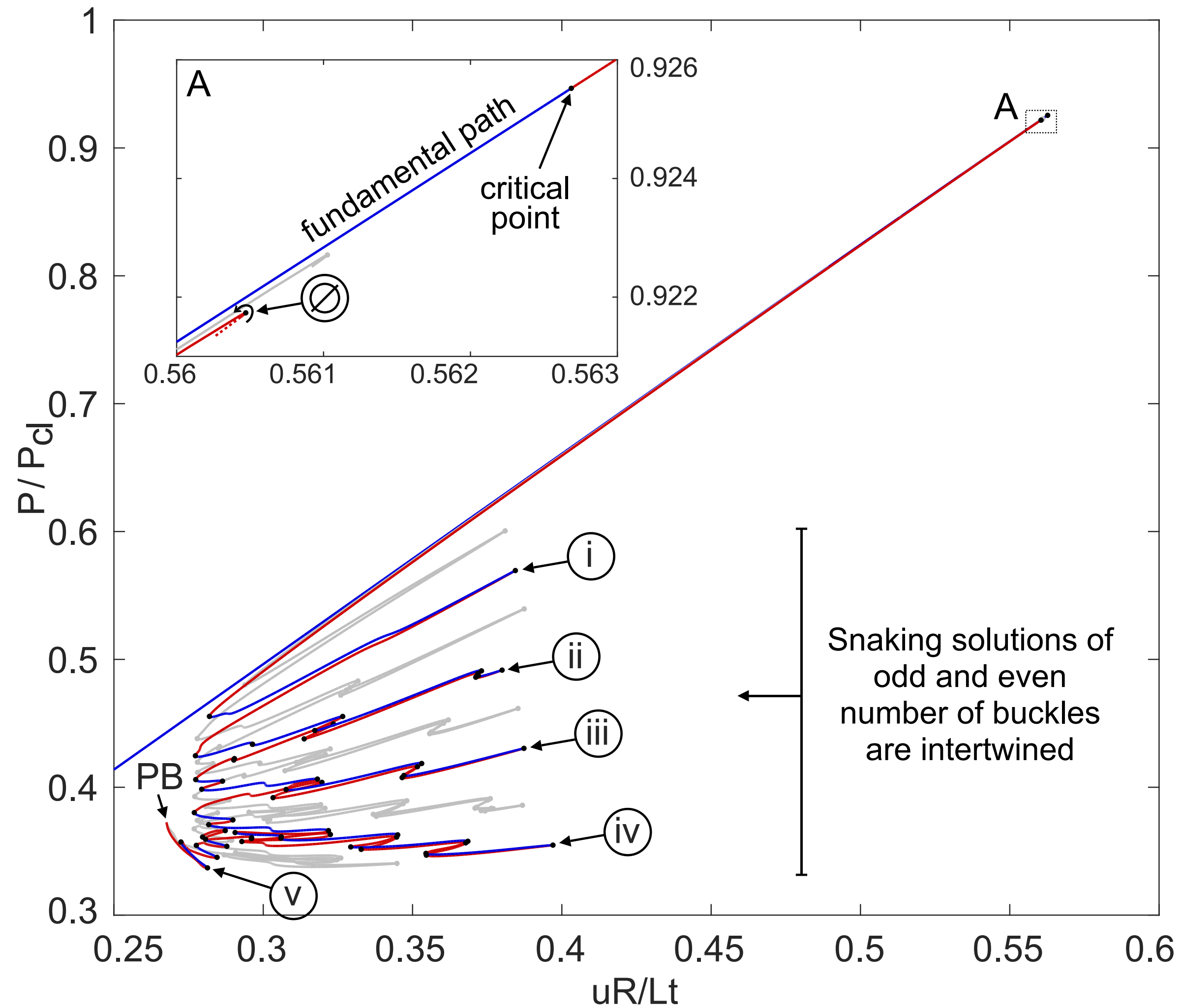


# Single-dimple Snaking – Odd Nr Buckles

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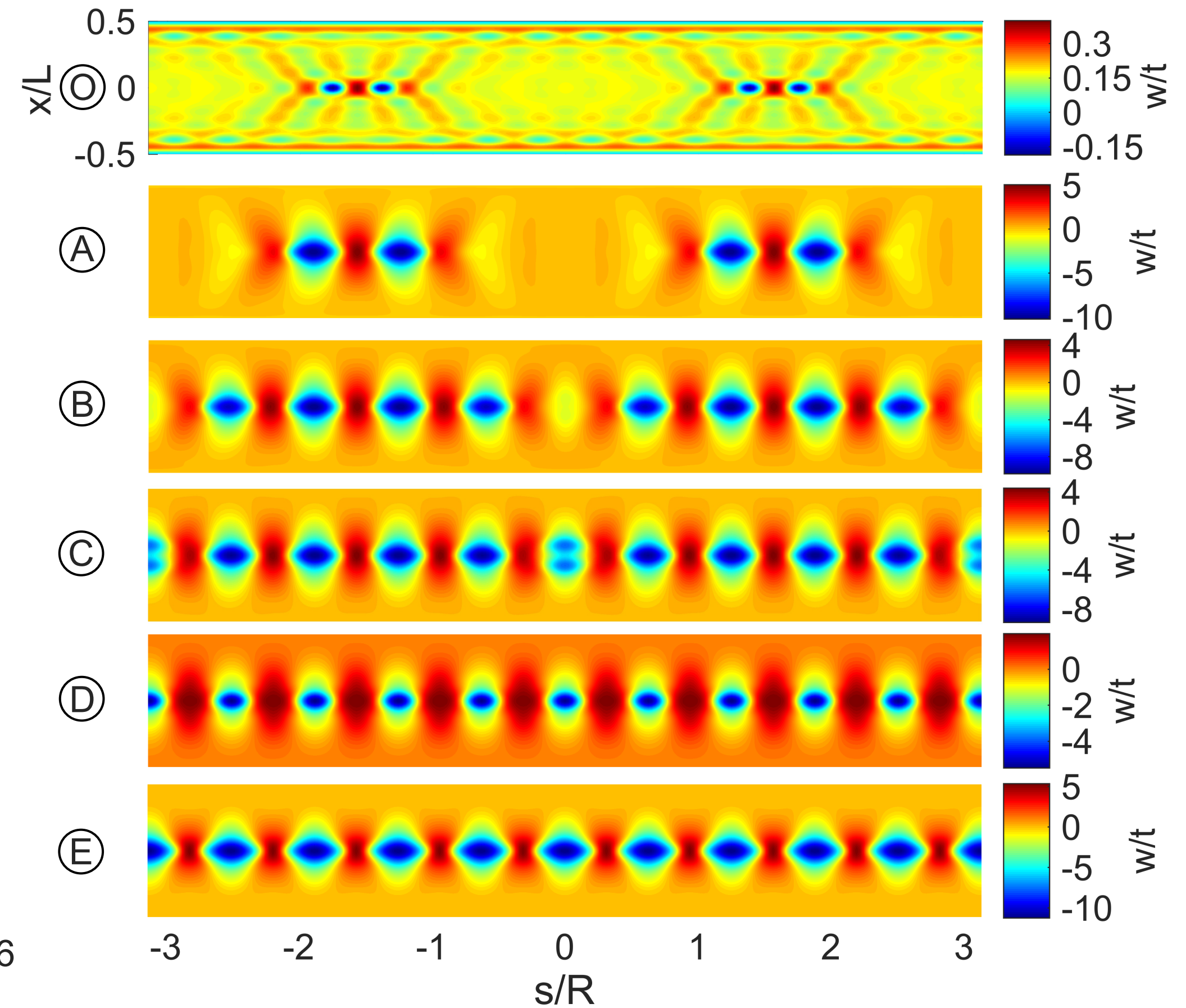
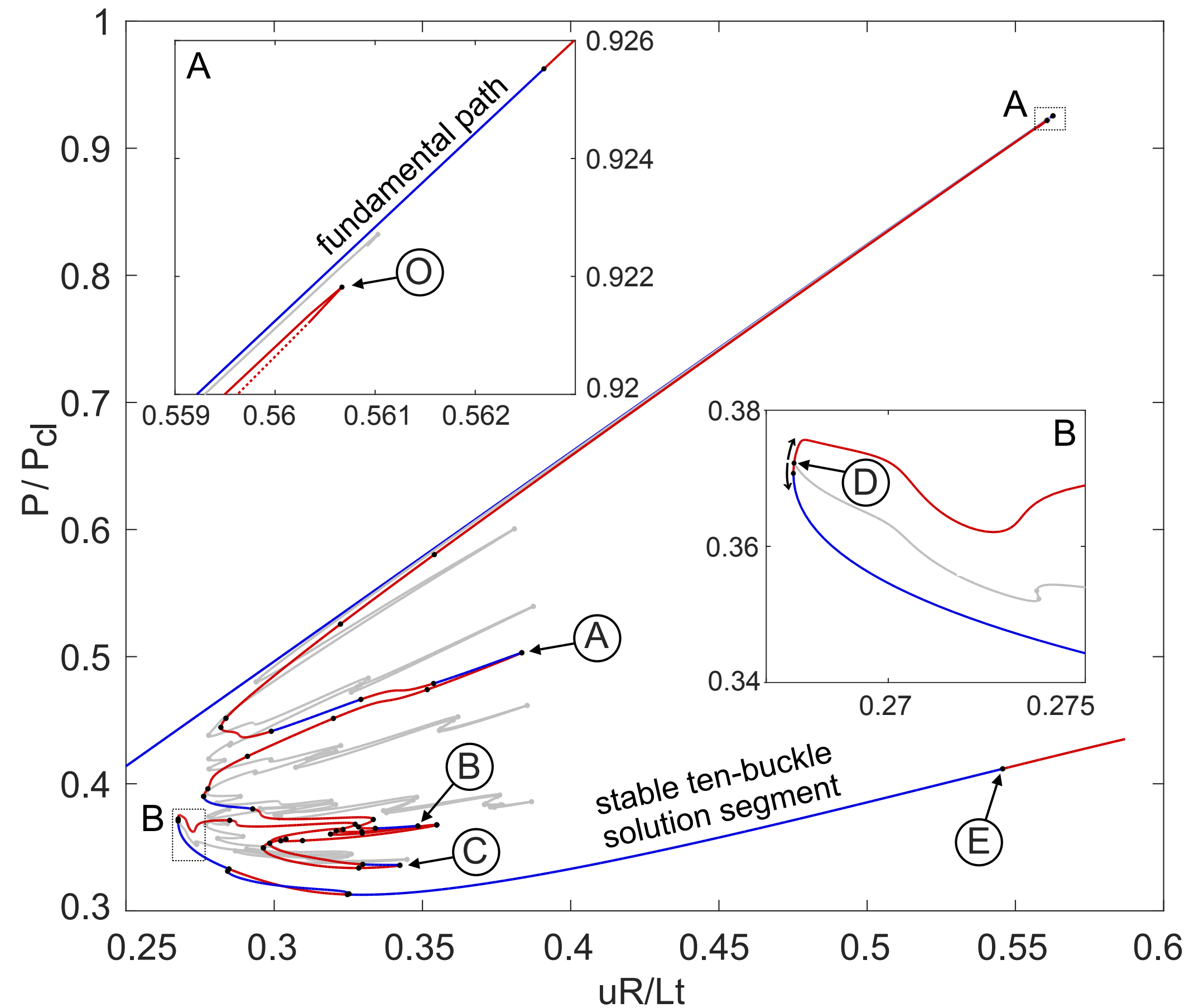
# Single-dimple Snaking – Even Nr Buckles





# Four-dimple Snaking

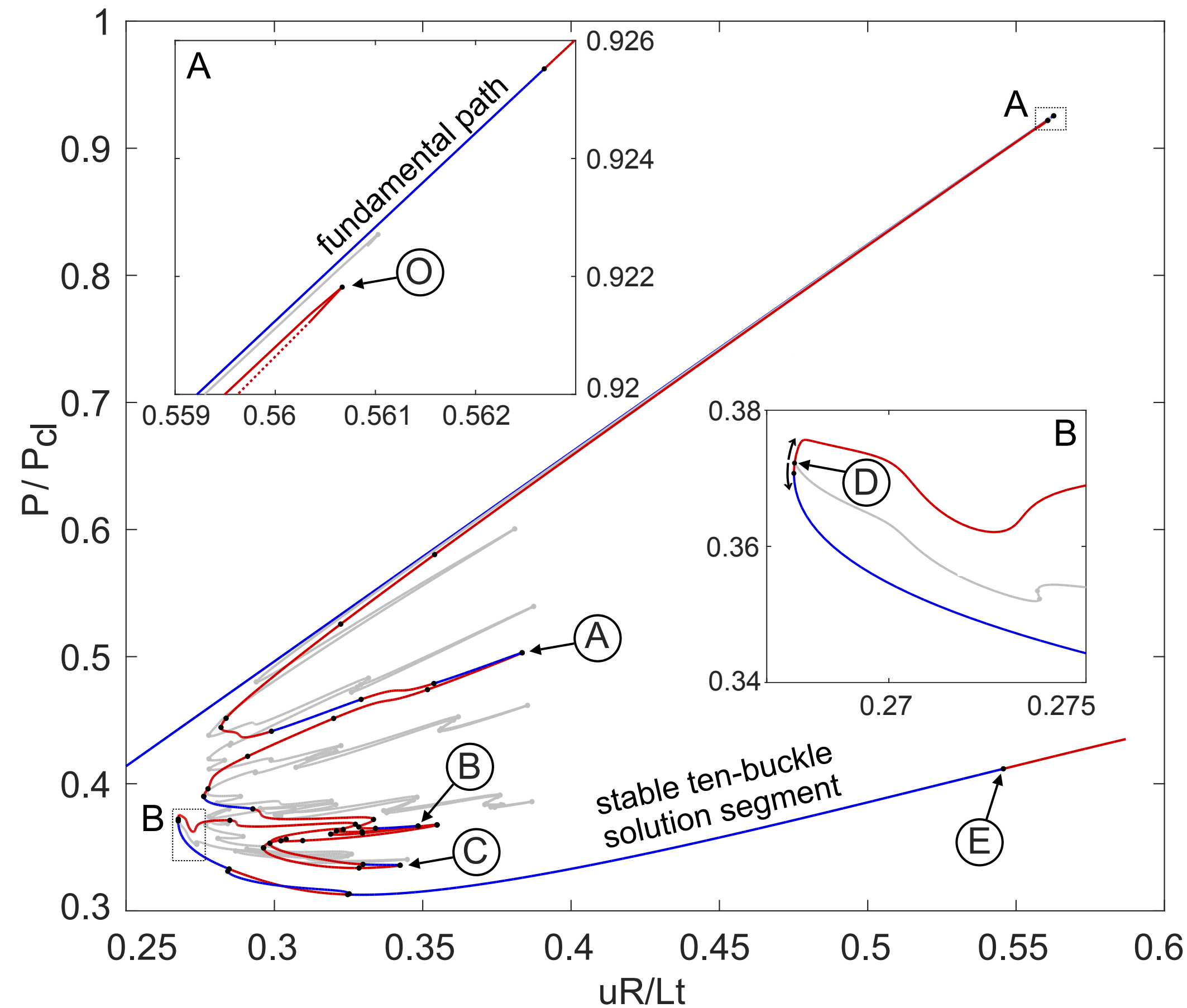
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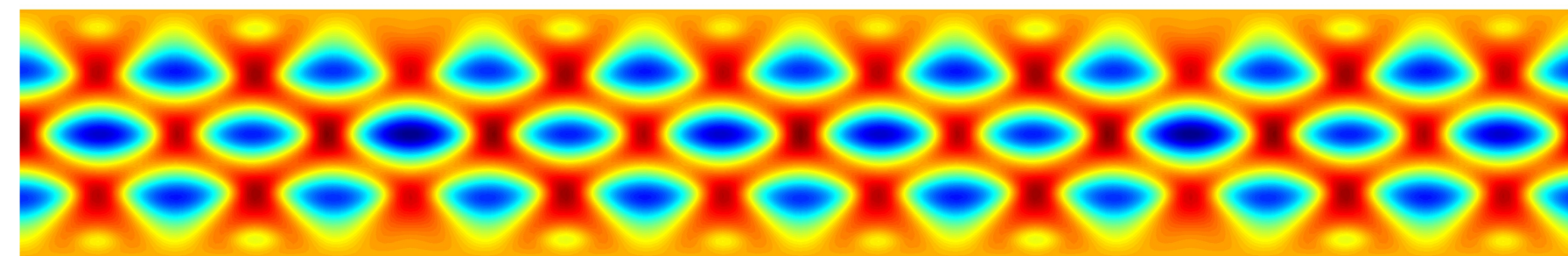
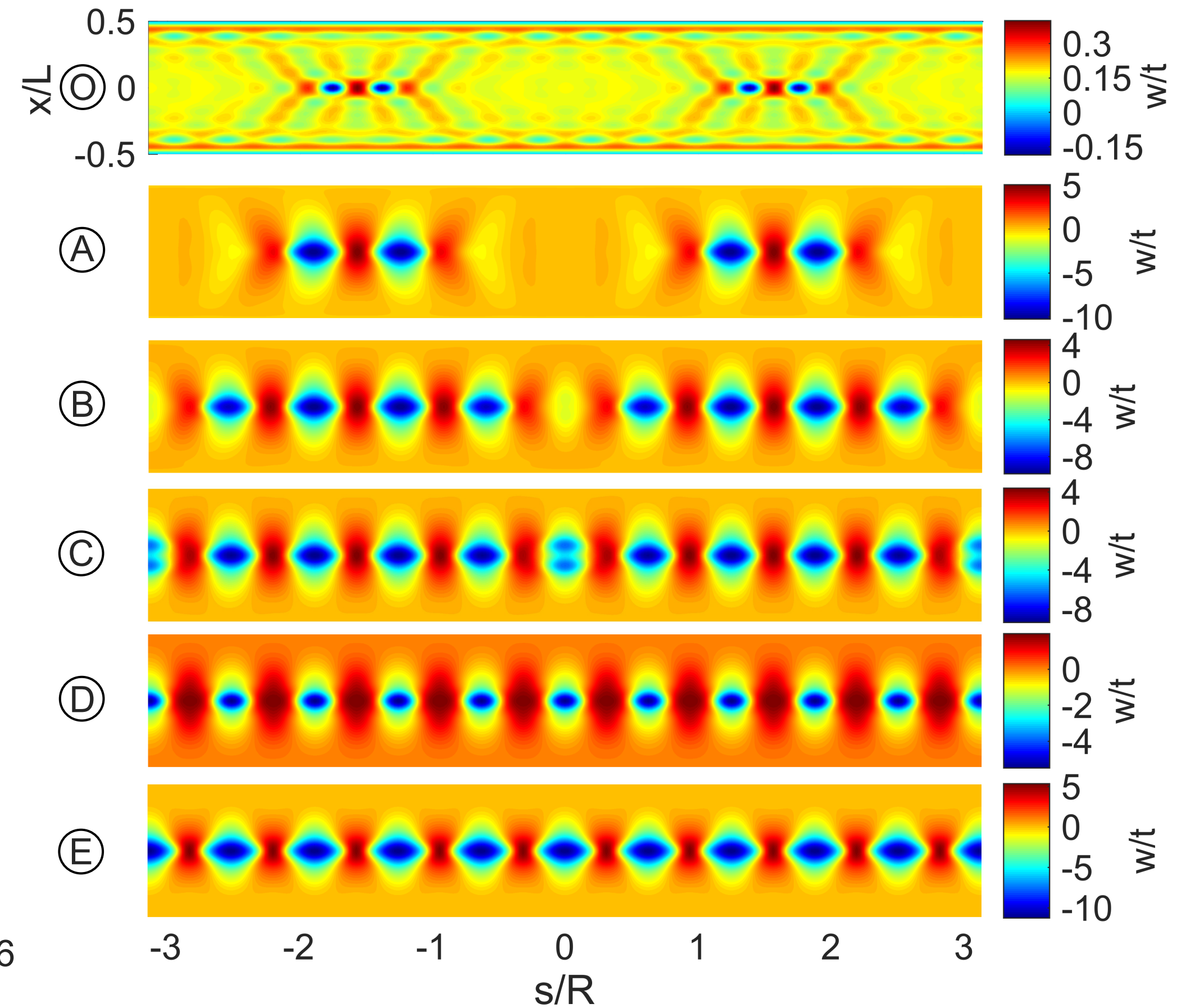


# Four-dimple Snaking

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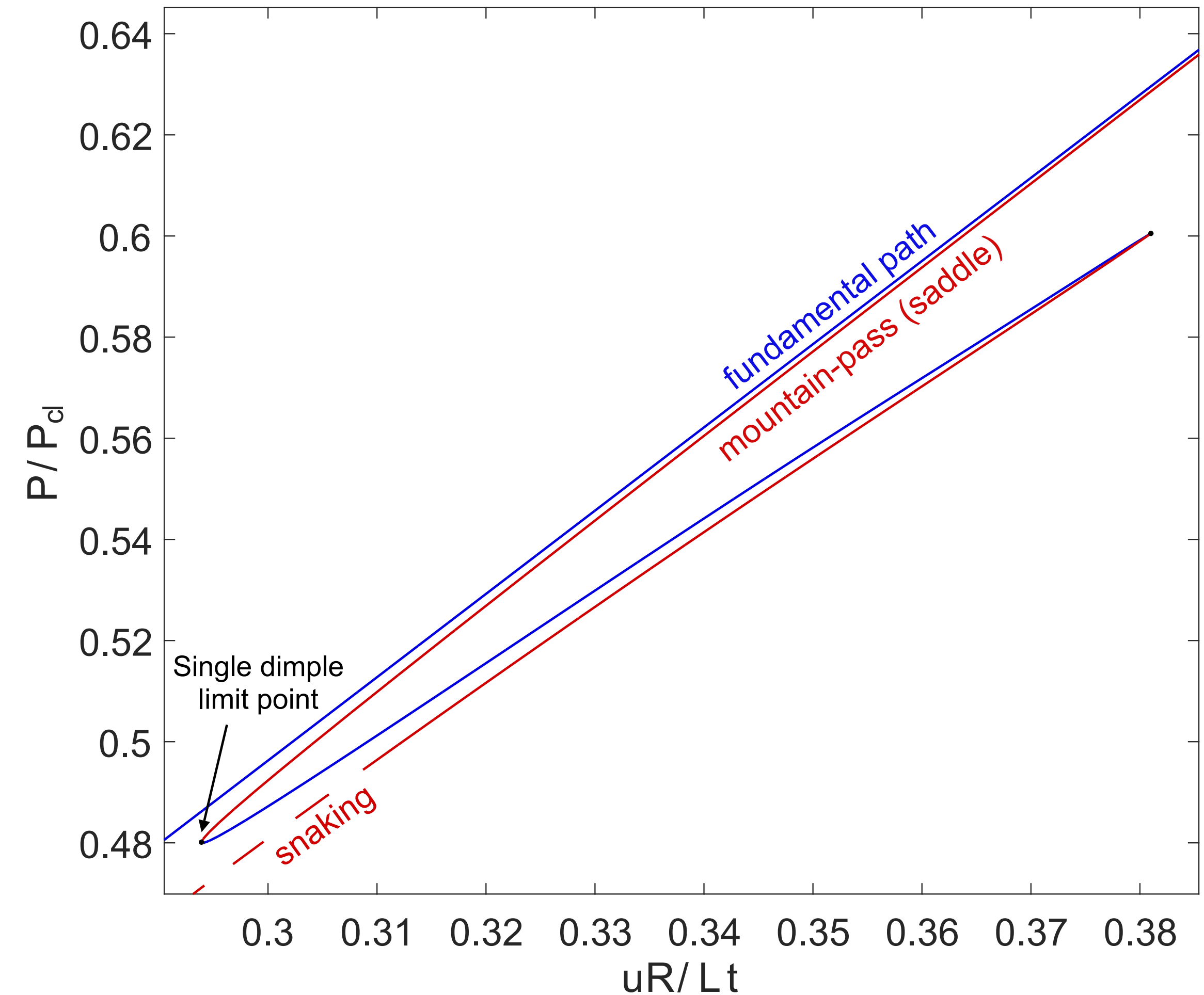
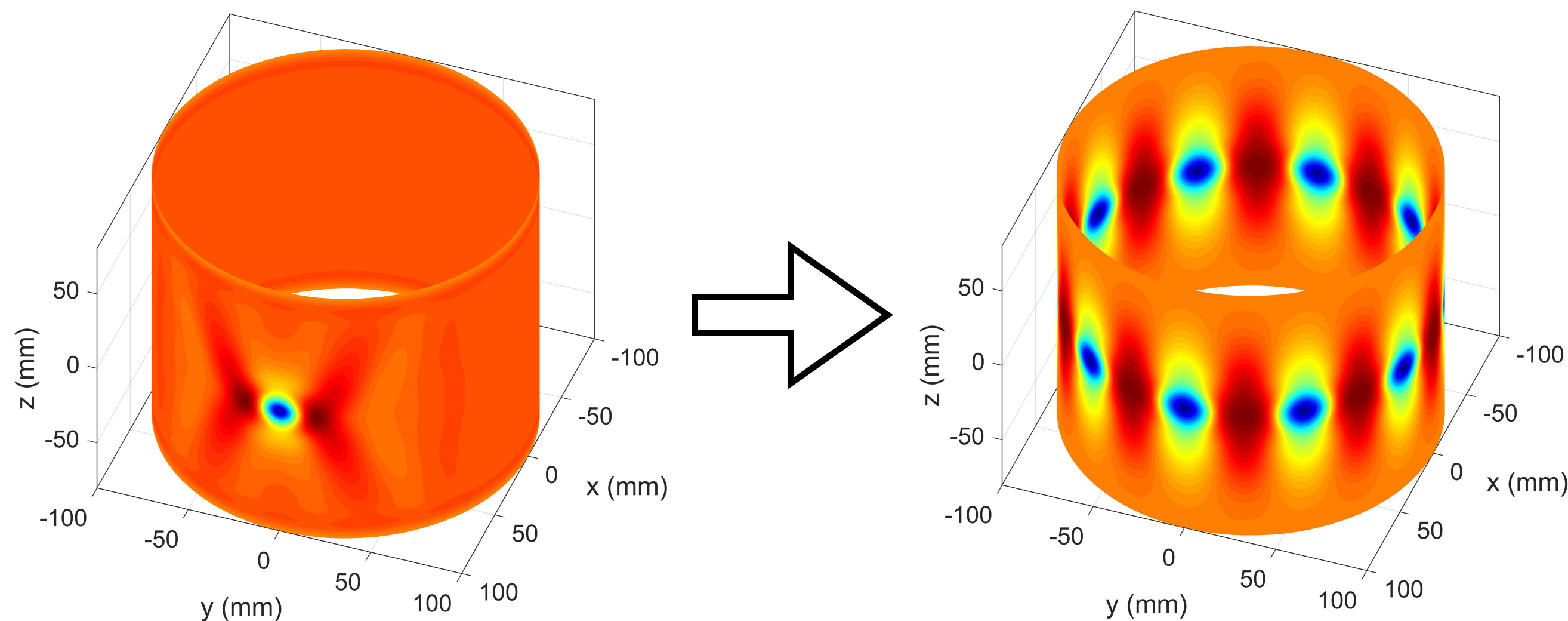
Yoshimura-like deep post-buckling pattern





# Single-dimple Tracking

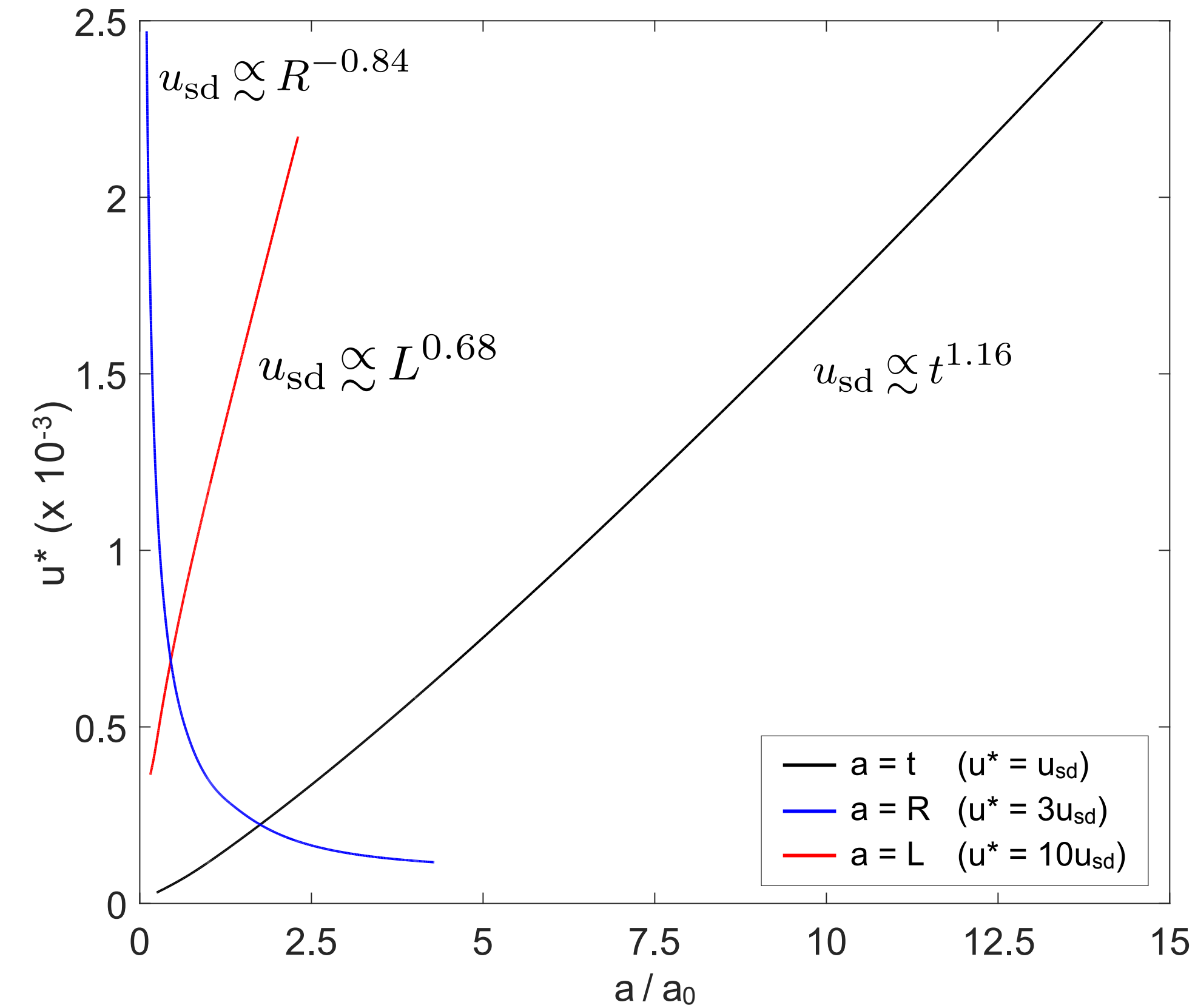
- From pre-buckling to stable diamond pattern via the single dimple
- When is the single dimple an unstable saddle?
- Use bifurcation tracking capability





# Single-Dimple Threshold

$R^2 > 0.995$  in all cases

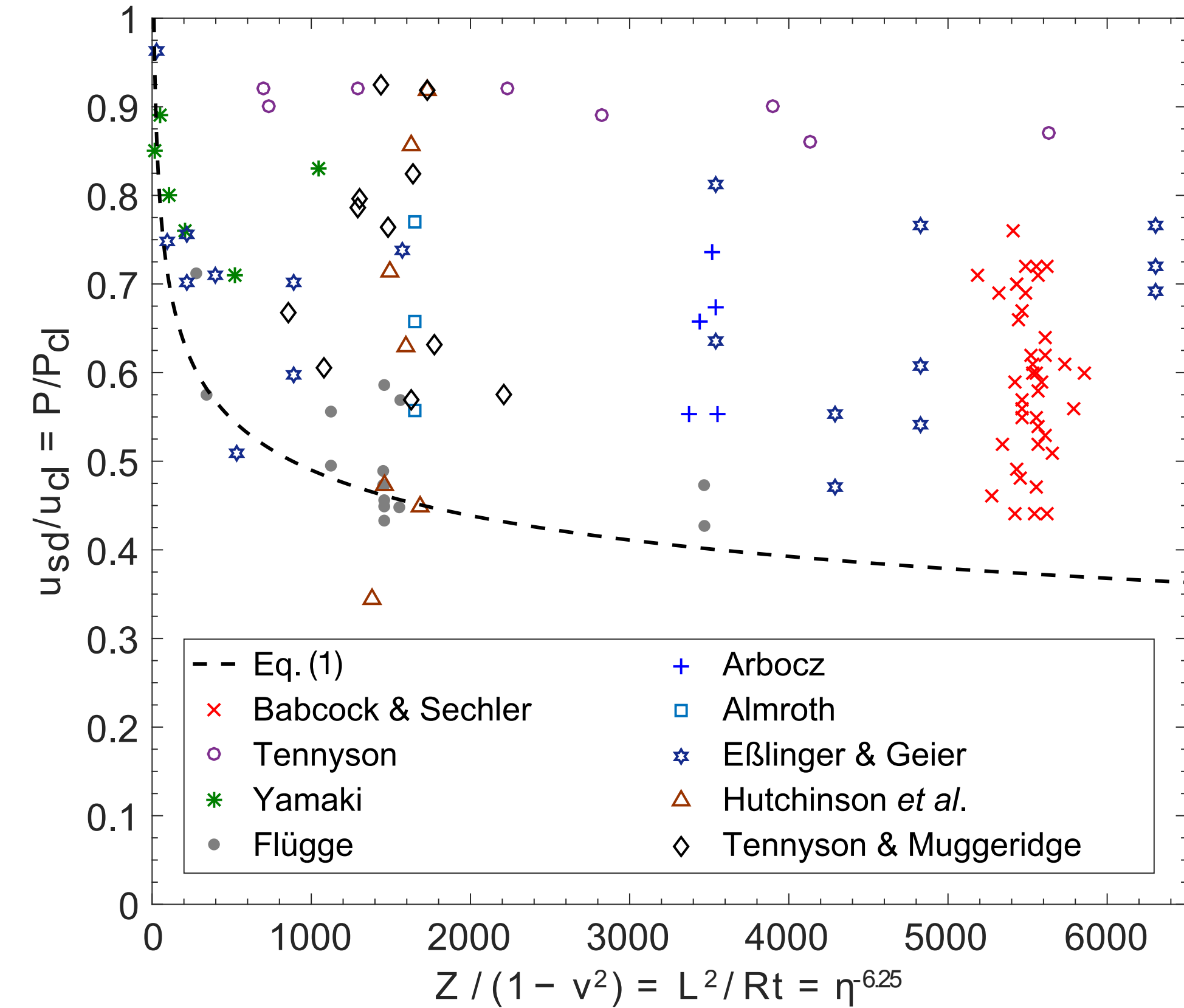


Single-dimple curve is  
a function of the  
Batdorf parameter

$$\frac{P}{P_{cl}} = 1.48 \frac{R^{0.16} t^{0.16}}{L^{0.32}} = 1.48 \eta$$

$$\frac{Z}{\sqrt{1 - \nu^2}} = \frac{L^2}{Rt} = \eta^{-6.25}$$

115 experimental results



# Comments

- Outlier by Hutchinson *et al.* (1971) is an axisymmetric dimple with relatively large amplitude
- Also, many data points from NASA SP-8007 exist below single-dimple curve, e.g. Donnell (1934)
- Asymptote NASA:  $P/P_{cl} \sim 0.2$  and SD:  $P/P_{cl} \sim 0.35$ 
  - previous manufacturing technology less controlled
- → Buckling below the threshold is possible, but not likely with modern engineering process control
- → Less conservative design guideline than NASA SP-8007 for cylinders manufactured to tight dimensional control

# On the role of localised post-buckling equilibria in axially compressed cylinders

Thank you for your attention!

# Single-Dimple Imperfections

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